Exercise 30

Calculate y'.

$$y = \frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}$$

Solution

Use logarithms to simplify the right side.

$$\begin{aligned}
\ln y &= \ln \frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5} \\
&= \ln(x^2+1)^4 - \ln[(2x+1)^3(3x-1)^5] \\
&= \ln(x^2+1)^4 - [\ln(2x+1)^3 + \ln(3x-1)^5] \\
&= \ln(x^2+1)^4 - \ln(2x+1)^3 - \ln(3x-1)^5 \\
&= 4\ln(x^2+1) - 3\ln(2x+1) - 5\ln(3x-1)
\end{aligned}$$

Now take the derivative of both sides with respect to x.

$$\frac{d}{dx}\ln y = \frac{d}{dx}[4\ln(x^2+1) - 3\ln(2x+1) - 5\ln(3x-1)]$$

$$\frac{1}{y}\cdot\frac{d}{dx}(y) = 4\left[\frac{d}{dx}\ln(x^2+1)\right] - 3\left[\frac{d}{dx}\ln(2x+1)\right] - 5\left[\frac{d}{dx}\ln(3x-1)\right]$$

$$\begin{aligned}
\frac{1}{y}\frac{dy}{dx} &= 4\left[\frac{1}{x^2+1}\cdot\frac{d}{dx}(x^2+1)\right] - 3\left[\frac{1}{2x+1}\cdot\frac{d}{dx}(2x+1)\right] - 5\left[\frac{1}{3x-1}\cdot\frac{d}{dx}(3x-1)\right] \\
&= 4\left[\frac{1}{x^2+1}\cdot(2x)\right] - 3\left[\frac{1}{2x+1}\cdot(2)\right] - 5\left[\frac{1}{3x-1}\cdot(3)\right] \\
&= \frac{8x}{x^2+1} - \frac{6}{2x+1} - \frac{15}{3x-1} \\
&= \frac{8x(2x+1)(3x-1) - 6(x^2+1)(3x-1) - 15(x^2+1)(2x+1)}{(x^2+1)(2x+1)(3x-1)} \\
&= \frac{(48x^3+8x^2-8x) - (18x^3-6x^2+18x-6) - (30x^3+15x^2+30x+15)}{(x^2+1)(2x+1)(3x-1)} \\
&= \frac{-x^2-56x-9}{(x^2+1)(2x+1)(3x-1)} \\
&= -\frac{x^2+56x+9}{(x^2+1)(2x+1)(3x-1)}
\end{aligned}$$

Therefore, multiplying both sides by y,

$$\begin{aligned}\frac{dy}{dx} &= -\frac{x^2+56x+9}{(x^2+1)(2x+1)(3x-1)}y \\ &= -\frac{x^2+56x+9}{(x^2+1)(2x+1)(3x-1)}\left[\frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}\right] \\ &= -\frac{(x^2+56x+9)(x^2+1)^3}{(2x+1)^4(3x-1)^6}.\end{aligned}$$